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PLASTIC DEFORMATION OF AN ISOTROPICALLY STRAIN-HARDENING POLYCRYSTALLINE MATERIAL

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UDC 539.374

It is known that the features of the elastoplastic deformation of metals are determined by their polycrystalline structure. Thus, the equations that describe the deformation of the polycrystal should be derived on the basis of study of the processes that take place within its grains. It has been established experimentally that, at moderate temperatures, plastic deformation occurs mainly by the mechanism of translational crystallographic slip. Slip is anisotropic and leads to strain-hardening of a single crystal. This strain-hardening is expressed in an increase in the limiting shear stress in both active (active strain-hardening) and passive (latent strain-hardening) systems and must be taken into account when choosing the corresponding strain-hardening law. The elastic and plastic anisotropy of crystallites and intergranular interactions occurring throughout the deformation history of the material cause the fields of local stresses and strains in it to be nonuniform. Thus, the equation that connects macroscopic stresses and strains should be determined by averaging relations between the corresponding local fields over the entire volume of the specimen. There is a fairly large number of consistent theories in which the researcher, in the course of deriving the governing equation, made a detailed study of one of the above-mentioned aspects of the plastic deformation of polycrystals [1-5].

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 148-153, September-October, 1991. Original article submitted November 10, 1989; revision submitted June 5, 1990.

0021-8944/91/3205-0796$12.50

In the present study, we propose a relatively simple method of obtaining an equation to describe the elastoplastic behavior of a material with an arbitrary loading program. In contrast to the well-established theories, we attempt to combine empirical substantiation of the local strain-hardening law with the concept of multiple slip and a relatively simple procedure for averaging over the space of orientations of the grains of the aggregate. A similar scheme was examined in [3] for uniaxial tension.

1. Plastic Deformation in Grains of a Polycrystal. We will examine one grain of a polycrystal undergoing plastic flow. Let the grain have n active slip systems. Then the following relation [2] should be satisfied inside the grain

$$t^{\alpha}_{ij} s_{ij} = \tau^{\alpha}, \quad \varepsilon_{ij} = 2t^{\alpha}_{ij}\lambda_{\alpha}. \tag{1.1}$$

Here, s_{ij}, ε_{ij} are the deviators of the local stresses and plastic strains, which are assigned in the laboratory coordinate system connected with the specimen; τ^{α}, λ_{α} are the shear stress and plastic shear acting in system α. The tensor of the Schmidt coefficients $t_{ij}{}^{\alpha}$ is determined by the expression

$$t^{\alpha}_{ij} = \frac{1}{2}\left[l^{(\alpha)}_i n^{(\alpha)}_j + l^{(\alpha)}_j n^{(\alpha)}_i\right] \tag{1.2}$$

($\mathbf{l}^{(\alpha)}$, $\mathbf{n}^{(\alpha)}$ are unit vectors in the slip direction and the direction of a normal to the slip plane of system α). No summation is performed over the indices in the brackets.

The strain-hardening law of the single crystal can be written in the form

$$\tau^{\alpha} = \tau_0 + \int H_{\alpha\beta}\, d\lambda_{\beta},$$

where $H_{\alpha\beta}$ are the coefficients of the strain-hardening matrix, these coefficients accounting for active and latent strain-hardening in slip systems and being dependent on the strain history of the material; τ_0 is the resolved shear stress associated with the initial shear. It has been observed experimentally that $H_{\alpha\beta}$ and τ_0 are dependent on the elastic properties and microstructure of the material [6].

It was shown in [7, 8] that τ_0 can be calculated with the Hall–Petch empirical equation for the macroscopic elastic limit of a polycrystalline material. It is determined by the type of crystalline lattice and mean grain size, but it is only slightly sensitive to the dispersion of the size distribution of the crystallites. The parameters $H_{\alpha\beta}$ characterize the strain-hardening of a single crystal in an elastoplastic medium and should be found from experiments conducted with polycrystalline specimens.

According to the isotropic strain-hardening law in [1]

$$\tau^{\alpha} = \tau_0 + \int H \sum_{\beta=1}^{n} d\lambda_{\beta}. \tag{1.3}$$

It follows from this that the flow stress is the same in all active systems. We will assume that the following expression [5] is sufficiently accurate for multiple slip in a crystal

$$\lambda_1 = \lambda_2 = \ldots = \lambda_n = \lambda, \tag{1.4}$$

This allows us, at each stage of the strain history, to introduce the concept of an effective slip system for plastic grains with the shear integral $\Lambda = n\lambda$ and the Schmidt coefficient tensor [5]:

$$t_{ij} = \frac{1}{n}\sum_{\alpha=1}^{n} t^{\alpha}_{ij}. \tag{1.5}$$

In (1.5), t_{ij} depends on the type and number of active slip systems in the crystal and its orientation relative to the loading axes.

Allowing for (1.3)-(1.5) and changing over from finite quantities to their increments, we find from (1.1) that

$$t_{ij}\dot{s}_{ij} = H\dot{\Lambda}, \quad \dot{\varepsilon}_{ij} = 2t_{ij}\dot{\Lambda}. \tag{1.6}$$

Here and below, $\dot{a} = da$.

We will examine materials which exhibit regular strain-hardening [9] in the region of activation of the slip systems in their grains. The stress–strain curves of such materials in this region are characterized by a monotonically decreasing strain-hardening modulus. Many metals and alloys can be classed as materials of this type.

We introduce the local strain-hardening modulus H through the relation

$$H = 2ht_{kl}t_{kl} \tag{1.7}$$

(h = const and depends on the type of material).

The parameter h in (1.7) is a discrete function of the number of active slip systems of the single crystal and decreases with an increase in n. Its value for each n depends on the type of macroscopic stress–strain state in the medium and the orientation of the grains relative to the loading axes. The representation of H in the form (1.7) models features of the regular strain-hardening of the medium at the microstructural level. For example, for an fcc crystal having 24 systems with four slip planes {111} and six slip directions [110], the strain-hardening modulus changes within the range H = h (1.0-0.534). Here, the left boundary of the interval corresponds to the exact upper boundary of H for n = 1, while the right boundary corresponds to its exact lower boundary for n = 5.

2. Relationship between Local and Macroscopic Stresses and Strains. In a plastically deformed polycrystal, it is possible to distinguish three structural phases formed by the grains at different stages of plastic deformation. The first of these phases (q = 1) is formed by crystals which are completely plastic. The number of active slip systems for these crystals is equal to the maximum possible number of independent systems ($n = N \leq 5$). The second phase (q = 2) is composed of crystallites which are not fully plastic ($n < N$), while the third phase (q = 3) is formed by elastic grains (n = 0). The spatial structure of such a polycrystal can be described by a set of random indicator functions of the coordinates $\kappa_q(\mathbf{r})$ (q = 1, 2, 3), each of which is equal to unity for the set of points of the q-th phase and equal to zero outside this set. Let v_1, v_2, v_3 be the volume concentrations of the corresponding crystallites in the material, dependent on its macroscopic stress–strain state.

Various models of intergranular interaction [1-5, 7] can be used to determine the connection between the local and macroscopic stress–strain states. Striving to obtain the simplest overall rheological equation, we use the results in [2]. We have

$$\begin{gathered}\dot{s}^{(q)}_{ij} = \dot{\eta}_{ij} - b\,\dot{\varepsilon}_{ij} \quad (q=1, 2, \quad \text{plastic grains}),\\ \dot{s}^{(3)}_{ij} = \dot{\eta}_{ij} \quad (q=3, \text{ elastic grains}).\end{gathered} \tag{2.1}$$

Combining Eqs. (2.1) by means of the indicator functions, we find

$$\dot{s}_{ij} = \dot{\eta}_{ij} - b\dot{\varepsilon}_{ij}\varkappa(\mathbf{r}) \ (\varkappa(\mathbf{r}) = \varkappa_1(\mathbf{r}) + \varkappa_2(\mathbf{r})). \tag{2.2}$$

Here

$$\dot{\eta}_{ij} = \beta\langle\dot{s}_{ij}\rangle + b\langle\dot{e}_{ij}\rangle, \ b = 2\mu(1-\beta), \ \beta = 2(4-5\nu)/15(1-\nu) \tag{2.3}$$

(μ is the shear modulus; ν is the Poisson's ratio; $\dot{e}_{ij}$ is the deviator of the tensor of the total local strains; $\langle a \rangle$ is the mathematical expectation of a).

Henceforth assuming that the random fields ε_{ij}, e_{ij}, s_{ij}, t_{ij}, κ_q are statistically uniform and ergodic, we replace their mathematical expectation by quantities, averaged over the volumes of the phases V_q and the total volume V, that coincide with the averages for the set of crystallite orientations in the phase and the medium, i.e.,

$$\langle a\rangle = \frac{1}{V}\int\limits_V a(\mathbf{r})\,d\mathbf{r}, \quad \langle a\rangle_q = \frac{1}{V_q}\int\limits_{V_q} a(\mathbf{r})\,d\mathbf{r}$$
$$(q = 1, 2, 3, \ V = V_1 + V_2 + V_3).$$

In this case, it is obvious that the equality $v_q = \langle\kappa_q(\mathbf{r})\rangle = V_qV^{-1}$ is valid.

Augmenting Eqs. (2.2) with Hooke's law for the local and macroscopic stresses and strains

$$\dot{s}_{ij} = 2\mu(\dot{e}_{ij} - \dot{\varepsilon}_{ij}\varkappa(\mathbf{r})),\ \langle\dot{\varepsilon}_{ij}\rangle = 2\mu(\langle\dot{e}_{ij}\rangle - \langle\dot{\varepsilon}_{ij}\rangle), \tag{2.4}$$

we obtain the complete system of relations which connects the local and macroscopic stress–strain states of the medium.

3. Equation of Plastic Strain of the Polycrystal. We will examine single crystals in the completely plastic state (q = 1). It follows from (1.6)-(1.7) that

$$t_{ij}\left[\dot{s}_{ij}^{(1)} - h\dot{\varepsilon}_{ij}^{(1)}\right] = 0. \tag{3.1}$$

The grains lose their plastic anisotropy when they are in the fully plastic state. Thus, condition (3.1) should be satisfied for any crystallographic direction. The following equation is valid as a result

$$\dot{s}_{ij}^{(1)} - h\dot{\varepsilon}_{ij}^{(1)} = 0. \tag{3.2}$$

We use (2.1) and (3.2) to determine the local strains in the first phase of the polycrystal

$$\dot{\varepsilon}_{ij}^{(1)} = \dot{\eta}_{ij}/(2\mu(1-\beta+\vartheta)). \tag{3.3}$$

As regards the partially plastic crystals of the second phase, we use (1.6)-(1.7) and (2.1) to obtain the below expression for integral shear

$$\dot{\Lambda}^{(2)} = \dot{\eta}_{ij}t_{ij}/(t_{kl}t_{kl}2\mu(1-\beta+\vartheta)). \tag{3.4}$$

We find the local plastic strains in the grains of the second phase from (1.6) and (3.4)

$$\dot{\varepsilon}_{ij}^{(2)} = \dot{\eta}_{kl}t_{kl}t_{ij}/(t_{pq}t_{pq}2\mu(1-\beta+\vartheta)). \tag{3.5}$$

Then averaging Eq. (2.2) over the entire volume of the material with allowance for (3.3) and (3.5) in accordance with the rule governing the mechanical mixing of phases, we find the macroscopic stresses in the medium

$$\begin{gathered}\langle\dot{s}_{ij}\rangle = \dot{\eta}_{ij} - b(\langle\dot{\varepsilon}_{ij}\rangle_1 + \langle\dot{\varepsilon}_{ij}\rangle_2) = (1 - v_1\varphi)\dot{\eta}_{ij} - v_2\varphi\Psi_{ijkl}\dot{\eta}_{kl},\\ \varphi = (1-\beta)/(1-\beta+\vartheta),\ \Psi_{ijkl} = \langle t_{ij}t_{kl}/t_{pq}t_{pq}\rangle_2.\end{gathered} \tag{3.6}$$

Expressing $\dot{\eta}_{ij}$ in (3.6) on the basis of (2.3), we represent the macroscopic equation describing the plastic deformation of the material in the form

$$\begin{gathered}\langle\dot{s}_{ij}\rangle = \chi_{ijkl}\langle\dot{e}_{kl}\rangle,\\ \chi_{ijkl} = 2\mu_*\left[I_{ijmn} + c(v_1)v_2\Psi_{ijmn}\right]^{-1}\left[I_{mnkl} - d(v_1)v_2\Psi_{mnkl}\right],\\ \mu_* = \mu a(v_1),\quad a(v_1) = [1-\beta+\vartheta-(1-\beta)v_1][1-\beta+\vartheta-\beta v_1]^{-1},\\ c(v_1) = \beta[1-\beta+\vartheta+\beta v_1]^{-1},\ d(v_1) = (1-\beta)[1-\beta+\vartheta-(1-\beta)v_1]^{-1},\\ I_{ijkl} = (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})/2.\end{gathered} \tag{3.7}$$

Here, the tensor of the elastoplastic shear moduli $\chi_{ijk\ell}$ depends on the distribution of active slip systems in the partially plastic grains and on the concentrations v_1 and v_2 of both plastic phases. In connection with this, its value at each loading stage is determined by the entire strain history of the material.

The macroscopic plastic strains of the medium are found from (2.4) and (3.7):

$$\begin{gathered}\langle\dot{\varepsilon}_{ij}\rangle = \Theta_{ijkl}\langle\dot{e}_{kl}\rangle,\\ \Theta_{ijkl} = I_{ijkl} - a(v_1)[I_{ijmn} - c(v_1)v_2\Psi_{ijmn}][I_{mnkl} - d(v_1)v_2\Psi_{mnkl}].\end{gathered} \tag{3.8}$$

Equations (3.7) and (3.8) completely determine the macroscopic stress–strain state of the medium. Equation (3.7) can be simplied for individual loading stages. In elastic deformation, the plastic phases are absent ($v_1 = v_2 = 0$) and the rheological equation coincides with Hooke's law $\langle \dot{s}_{ij}\rangle = 2\mu\langle \dot{e}_{ij}\rangle$.

In the region of regular strain-hardening, all three phases are present to the same extent in the material ($v_1 \neq v_2 \neq v_3 \neq 0$). The strain equation has the form (3.7) in this case. With further monotonic loading, grains in the fully plastic state begin to predominate in the polycrystal ($v_1 \sim 1$, while v_2 and v_3 are small). Expanding $\chi_{ijk\ell}$ into a series in $v_2 \ll 1$ and keeping terms of orders no greater than v_2, we obtain the approximate equality

$$\chi_{ijkl} = 2\mu_* \; [I_{ijkl} - v_2 f(v_1) \Psi_{ijkl}], \quad f(v_1) = c(v_1) + d(v_1), \tag{3.9}$$

which is valid at the end of the transitional region.

After completion of the period of regular strain-hardening, the material reaches the fully plastic state $v_1 = 1$, $v_2 = 1$. Here, we find from (3.7) and (3.9) that

$$\langle \dot{s}_{ij} \rangle = 2\mu\vartheta(1 + \vartheta)^{-1} \langle \dot{\varepsilon}_{ij} \rangle, \tag{3.10}$$

from which it follows that at $\vartheta = 0$ $\langle \dot{s}_{ij} \rangle = 0$ – at the fully plastic stage, a material with non-strain-hardening grains behaves as an ideally plastic material. With an unlimited increase in ϑ, the behavior of the medium is close to elastic. Finally, at $0 < \vartheta < \infty$, when fully plastic the polycrystal undergoes linear strain-hardening with the modulus $\mu\vartheta(1 + \vartheta)^{-1}$.

The fully plastic state is unattainable for the small group of materials in which the rate of isotropic strain-hardening of potentially active slip systems exceeds the rate of increase in the resolved shear stresses due to an increase in external stresses. In this case, asymptotic formulas (3.9) and (3.10) are no longer valid.

4. Special Types of Loading. The Parameter ϑ. Let us examine the tension of a polycrystalline specimen by the stresses $\langle \sigma_{11} \rangle \neq 0$. It follows from (3.6) that

$$\langle \dot{s}_{11} \rangle = (1 - v_1\varphi)\dot{\eta}_{11} - v_2\varphi\Psi_{11kl}\dot{\eta}_{kl}, \tag{4.1}$$

where $\dot{\eta}_{22} = \dot{\eta}_{33} = -\dot{\eta}_{11}/2$; $\dot{\eta}_{ij} = 0$ $(i \neq j)$.

Taking into account that $\psi_{11pp} = 0$, we find from (4.1) that

$$\langle \dot{s}_{11} \rangle = [1 - \varphi(v_1 + v_2\psi_p)]\dot{\eta}_{11} \; (\psi_p = 3\Psi_{1111}/2). \tag{4.2}$$

The following relations are valid for the given stress–strain state

$$\langle \dot{s}_{11} \rangle = 2\langle \dot{\sigma}_{11} \rangle/3, \; \langle \dot{e}_{11} \rangle = 2(1 + \nu)\langle \dot{\gamma}_{11} \rangle/3 \tag{4.3}$$

($\langle \gamma_{11} \rangle$ is a component of the tensor of the total macroscopic strains). With allowance for (2.3) and (4.3), we find from (4.2) that

$$\begin{gathered} \langle \dot{\sigma}_{11} \rangle = E_\mathrm{s}\langle \dot{\gamma}_{11} \rangle, \; E_\mathrm{s} = E\delta(v_1, \, v_2, \, \psi_p), \\ \delta(v_1, \, v_2, \, \psi_p) = [\vartheta + (1 - \beta)(1 - v_1 - v_2\psi_p)][1 + \vartheta - \beta(1 - v_1 - v_2\psi_p)]^{-1} \end{gathered} \tag{4.4}$$

(E is the Young's modulus).

In the case of pure shear ($\langle \sigma_{12} \rangle \neq 0$), by similar reasoning we arrive at

$$\langle \dot{\sigma}_{12} \rangle = 2\mu_\mathrm{s}\langle \dot{\gamma}_{12} \rangle, \; \mu_\mathrm{s} = \mu\delta(v_1, \, v_2, \, \psi_c), \; \psi_c = 2\Psi_{1212}. \tag{4.5}$$

For materials in which complete plasticity is possible, Eqs. (4.4) and (4.5) make it possible to determine the dimensionless strain-hardening parameter ϑ from experimental stress–strain curves of polycrystalline specimens tested in tension or torsion. In the case of tension, for example, if we assume in (4.4) that $v_1 = 1$ and $v_2 = 0$ we obtain

$$E_\mathrm{s} = E\vartheta(1 + \vartheta)^{-1}. \tag{4.6}$$

For the experimental stress–strain curve, $E_\mathrm{s} = d\langle \sigma_{11} \rangle / d\langle \gamma_{11} \rangle$ is the shear modulus of the curve beyond the limits of the transitional section. We conclude from (4.6) that $\vartheta = \omega(1 - \omega)^{-1}$ ($\omega = E_\mathrm{s}E^{-1}$).

Let us compare the analytical relations obtained for tension and torsion (4.4) and (4.5) (lines) with experimental data (x's in Fig. 1) from [10] for an aluminum alloy with an fcc lattice. In accordance with [10], the elastic limit of the material in tension

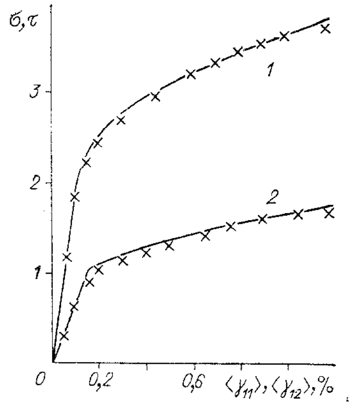

Fig. 1

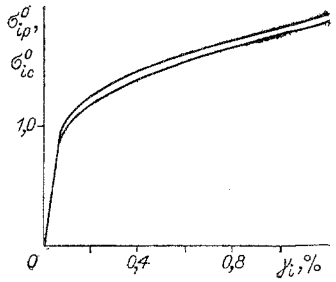

Fig. 2

$\sigma_s = 0.372$ N/m², the Young's modulus $E = 7.07 \cdot 10^2$ N/m², and the shear modulus $\mu = 2.65 \cdot 10^2$ N/m². With allowance for (1.1), we took the initial shear stress to be equal to $\tau_0 = \sigma_s/2 = \tau_s$ (τ_s is the elastic limit of the material in shear). We plotted the following quantities off the coordinate axis: $\sigma = \langle\sigma_{11}\rangle/\tau_0$, $\langle\gamma_{11}\rangle$ – curve 1 (tension) and $\tau = \langle\sigma_{12}\rangle/\tau_0$, $2\langle\gamma_{12}\rangle$ – curve 2 (pure shear). The parameter ω, determined from the experimental tension curve, was 0.044. This value corresponds to $\vartheta = 0.046$. In calculating the uniaxial tension curve from Eq. (4.4), we determined the parameter ψ_p by averaging over the set of orientations $m = 91$ within the first crystallographic triangle [2].

To calculate the pure shear curves from Eq. (4.5), we determined the number of crystal orientations in the computation of ψ_C as $M = m \times p = 280$ ($m = 28$ is the number of points within the crystallographic triangle, $p = 10$ is the number of crystal orientations about the fixed axis). The curves in Fig. 1 illustrate the satisfactory agreement between the theoretical and experimental data for the two cases examined.

Figure 2 shows curves obtained when the theoretical curves plotted in Fig. 1 are recalculated in the space of stress and strain intensities in accordance with the formulas

$$\sigma_i = \frac{1}{3}\sqrt{((\langle\sigma_{11}\rangle - \langle\sigma_{22}\rangle)^2 + (\langle\sigma_{11}\rangle - \langle\sigma_{33}\rangle)^2 + ((\langle\sigma_{22}\rangle) - \langle\sigma_{33}\rangle)^2 + 6(\langle\sigma_{12}\rangle^2 + \langle\sigma_{13}\rangle^2 + \langle\sigma_{23}\rangle^2)},$$

$$\gamma_i = \frac{1}{3}\sqrt{((\langle\gamma_{11}\rangle - \langle\gamma_{22}\rangle)^2 + (\langle\gamma_{11}\rangle - \langle\gamma_{33}\rangle)^2 + (\langle\gamma_{22}\rangle - \langle\gamma_{33}\rangle)^2 + 6(\langle\gamma_{12}\rangle^2 + \langle\gamma_{13}\rangle^2 + \langle\gamma_{23}\rangle^2)}. \quad (4.7)$$

The values of γ_i are plotted off the x-axis, while $\bar{\sigma}_i^0 = \sigma_i/\tau_0$ is plotted off the y-axis.

We find from (4.4)-(4.5) that $\dot{\sigma}_{ip} = E_{ip}\dot{\gamma}_{ip}$, $E_{ip} = 2\mu\delta(v_1, v_2, \psi_p)$, $\dot{\sigma}_{iC} = E_{iC}\dot{\gamma}_{iC}$, $E_{iC} = 2\mu\delta(v_1, v_2, \psi_C)$.

Analytical curve 1 in Fig. 2, corresponding to pure shear, is somewhat lower than curve 2 for uniaxial tension. Here, at the stage of complete plasticity ($v_1 = 1$, $v_2 = 0$) $E_{ip} = E_{iC} = \mu\vartheta(1 + \vartheta)^{-1}$.

Thus, the condition expressing the situation in which the relation σ_i^0–γ_i is independent of the type of stress–strain state is not satisfied for the proposed mathematical model of a polycrystalline medium. The calculations show a deviation from this condition in the same direction as indicated by experiments that have been completed on many materials [11].

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DYNAMIC DAMAGE AND FRACTURE OF A PLATE WITH THE EXPANSION OF A GAS CAVITY IN WATER

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UDC 539.3

A large number of studies (see [1], for example) have examined the fracture of a plate by pressure created in water by a gas bubble. The studies conducted thus far have generally dealt with explosive loads. In the present investigation, we examine low pressures in the bubble ($\Delta p \gtrsim 10^5$ Pa) which are created when gas is discharged from a high-pressure chamber. There are actually no small parameters in the problem in this cae, and study of the plate's fracture requires allowance for the effect of the plate on the expanding bubble, the character of discharge from the chamber, the stress—strain state of the plate, and damage accumulation in the plate.

1. Formulation of the Problem. We will examine a plate of thickness h lying on the surface of a liquid of semiinfinite depth (Fig. 1). A gas cavity located at the depth H begins to expand at a certain moment of time. The excess pressure Δp causes the plate to deform and crack, resulting in the formation of a hole of radius r^*. This very complex problem will be analyzed in two stages. First we study a linear model of the deformation of an infinite plate. The solution of this problem gives us the pressure distribution on the surface of the plate p(r, t). In the second stage, we use p(r, t) to calculate the fracture of a plate of finite dimensions with allowance for nonlinear strains.

Let us state the main assumptions underlying the given model: 1) the material remains elastic until fracture; 2) the characteristic length of the wave in the plate is much greater than its thickness; 3) the liquid is incompressible and ideal and the flow is a potential flow; 4) the gas cavity is spherical.

Given these assumptions, the equations of the liquid, with boundary conditions for the plate and the bubble, have the form

$$\Delta\varphi=0 \quad \text{at} \quad z<0, \quad \frac{\partial\zeta}{\partial t}+\frac{\partial\zeta}{\partial r}\frac{\partial\varphi}{\partial r}+\frac{1}{r^2}\frac{\partial\zeta}{\partial\alpha}\frac{\partial\varphi}{\partial\alpha}=\frac{\partial\varphi}{\partial z} \quad \text{at} \quad z=\zeta,$$
$$\rho\frac{\partial\varphi}{\partial t}+\rho\frac{v^2}{2}+\rho g\zeta+D\Delta_{\perp}^2\zeta+\rho_w h\frac{\partial^2\zeta}{\partial t^2}=0 \quad \text{at} \quad z=\zeta, \quad \frac{\partial\varphi}{\partial r}=u_0\cos\theta+\dot{a} \quad \text{at} \quad R=a, \tag{1.1}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 154-158, September-October, 1991. Original article submitted January 17, 1990.

0021-8944/91/3205-0802$12.50